HEAT TRANSFER FROM A DISC ROTATING IN A HOUSING WITH A RADIAL FLOW OF COOLANT

V. M. Kapinos

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An approximate solution, more general than that in [1], has been obtained for heat transfer from a disc with radial flow of coolant from center to periphery. The theory is compared with experimental data.

This paper is devoted to the problem studied in [1], namely, flow of a cooling agent from axis to periphery in the gap between a disc and its housing in the absence of an initial swirl. Attention is restricted to the part of the channel in which core flow exists, i.e., where the sum of the boundary-layer thicknesses at disc and housing is less than the gap width, or in the extreme case equal to the gap width at the end section. Then the circumferential component of the core flow velocity will be zero by virtue of the potential nature of the flow and the specified condition of zero swirl at the gap inlet.

The profiles of the circumferential and radial velocity components in the turbulent boundary layer at the disc may be written in the form:

$$V_\varphi = r\omega\left[1-\left(\frac{z}{\delta}\right)^{\frac{1}{7}}\right],\quad V_r=\left(\frac{z}{\delta}\right)^{\frac{1}{7}}\left[c\,\omega r\left(1-\frac{z}{\delta}\right)+V_{r_0}\right],$$
$$V_{r_0}=\frac{Q}{2\pi rs}. \tag{1}$$

The profiles satisfy the boundary conditions: $z = 0$, $V_\varphi = r\omega$, $V_r = 0$; $z = \delta$, $V_\varphi = 0$, $V_r = V_{r_0}$.

For large flows, according to the measurements of [2], the radial velocity component profile is convex, resembling that in a tube. This case, for which $c\omega r \ll V_{r_0}$, was examined by the author in [1]. At small flows, one would expect a radial velocity profile near the disc of the type obtained for flow on a freely rotating disc [3], or on a disc rotating in a housing without mass flow [4, 5, 6]. This is allowed for in that, when $V_{r_0} \to 0$, the profiles (1) coincide with the profiles V_φ and V_r adopted by von Karman [3] for a freely rotating disc. In accordance with the solution of [3], and in sufficiently good agreement with the experimental data of [7], c will be taken to be equal to 0.162. A similar radial velocity component profile was used in [8].

Inserting (1) in the boundary layer momentum equation

$$\rho\frac{d}{dr}\left(r^2\int_0^\delta V_rV_\varphi\,dz\right)=-\tau_\varphi r^2, \tag{2}$$

and taking the same friction stresses as for a plate, we have

$$\tau_\varphi=-0{,}0225\,\rho V_0^{7/4}\nu^{1/4}\delta^{-1/4}\cos\beta,\ V_0=\omega r[1+(c+K_v^{-1}x^{-2})]^{0{,}5},$$
$$\cos\beta=[1+(c+K_v^{-1}x^{-2})^2]^{-0{,}5},\ K_v=2\pi r_0^2\frac{\omega s}{Q},\quad x=\frac{r}{r_0}. \tag{3}$$

Then the boundary layer thickness may be determined from

$$\frac{d}{dx}\left[\delta\left(0{,}0681\,cx^4+0{,}0972\frac{x^2}{K_v}\right)\right]=$$
$$=0{,}0225x^{\frac{15}{4}}r_0^{\frac{3}{4}}\left[1+\left(c+\frac{1}{K_vx^2}\right)^2\right]^{3/8}\left(\frac{\nu}{\omega\delta}\right)^{1/4}. \tag{4}$$

The integral of this equation with initial conditions $r = r_0$, $\delta = 0$, has the form

$$\frac{\delta}{r} = 0.493\frac{K_v a^{0.8}}{\mathrm{Re}_L^{0.2}x^{2.6}(cK_v x^2 + 1.428)}, \quad \mathrm{Re}_L = \frac{\omega r^2}{\nu},$$

$$a = \int_1^x x^{4.75}\left[1 + \left(c + \frac{1}{K_v x^2}\right)^2\right]^{0.375}\left(c + \frac{1.428}{K_v x^2}\right)^{0.25} dx. \tag{5}$$

Combining (3) and (5), the dimensionless friction stress on the disc is:

$$\frac{\tau_\varphi}{\rho(r\omega)^2} = 0.0268x^{1.15}\left[1 + \left(c + \frac{1}{K_v x^2}\right)^2\right]^{0.375}\left(c + \frac{1.428}{K_v x^2}\right)^{0.25} \Big/ \mathrm{Re}_L^{0.2}a^{0.2}. \tag{6}$$

Using (5), we can also determine the limiting value of Re_L, up to which profiles (1) are applicable. It is known that the 1/7 power law applies to flow on a plate if $\mathrm{Re} = \omega\delta/\nu \leq 10^5$. Assuming $w = \omega r$, and defining δ according to (5), we have

$$\mathrm{Re}_L \leqslant 4{,}3\cdot 10^6\frac{x^{3.25}(cK_v x^2 + 1.428)^{1.25}}{K_v^{1.25}a}. \tag{7}$$

The local heat transfer coefficient may be determined by means of the Reynolds analogy. In the case dealt with here, the relationship between the dimensionless coefficients of heat transfer and friction is given by

$$\mathrm{Nu}_L = \frac{\tau_\varphi}{\rho(r\omega)^2}\mathrm{Re}_L, \quad \mathrm{Nu} = \frac{\alpha r}{\lambda}, \tag{8}$$

if the temperature distribution over the disc radius obeys the quadratic law $t_d - t_0 = kr^2$ and $\mathrm{Pr} = 1$ [8, 1]. The simultaneous solution of (6) and (8) gives

$$\mathrm{Nu}_L = 0.0268\,\mathrm{Re}_L^{0.8}A_L(K_v x),$$

$$A_L(K_v, x) = \frac{[c + (1 + 1/K_v x^2)^2]^{0.375}(c + 1.428/K_v x^2)^{0.25}x^{1.15}}{a^{0.2}}. \tag{9}$$

The average value of the Nusselt number is, by definition,

$$\mathrm{Nu} = R\frac{\int_{r_0}^{R}\mathrm{Nu}_L(t_d - t_0)\,dr}{\int_{r_0}^{R}(t_d - t_0)\,rdr}, \quad \mathrm{Nu} = \frac{\alpha R}{\lambda}. \tag{10}$$

Performing the integration, we have

$$\mathrm{Nu} = 0.134\,\mathrm{Re}^{0.8}A(K_v, X), \quad A(K_v, X) = \frac{a^{0.8}}{X^{0.6}(X^4 - 1)},$$

$$\mathrm{Re} = \frac{\omega R^2}{\nu}, \quad X = \frac{R}{r_0}. \tag{11}$$

When $c = 0$, and $1/K_V x^2 \ll 1$, we have, from (9) and (10),

$$\mathrm{Nu}_L^0 = 0.040\frac{\mathrm{Re}_L^{0.8}x^{0.65}}{K_v^{0.2}(x^{5.25} - 1)^{0.2}}, \quad \mathrm{Nu}^0 = 0.038\frac{\mathrm{Re}^{0.8}(X^{5.25} - 1)^{0.8}}{K_v^{0.2}X^{0.6}(X^4 - 1)}. \tag{12}$$

Analogous formulas were obtained in [1]. However, the coefficients in the formulas of [1], where an allowance was made for the difference between the average and maximum values of V_r, are 4% higher.

Graphs of the functions $A_L(K_V, x)$, $A(K_V, X)$ are given in Fig. 1. In addition, values of the correction function $\varepsilon = \mathrm{Nu}/\mathrm{Nu}^0$ are given in Fig. 2, for convenience of comparison with experimental data and evaluation of the region of application of formulas of type (12) in terms of the parameters K_V and X. It may be seen from Fig. 2 that ε is close to unity in the region of large X and K_V in the range 0.5-2.0.

It is of interest to compare approximations (11) with experimental data.

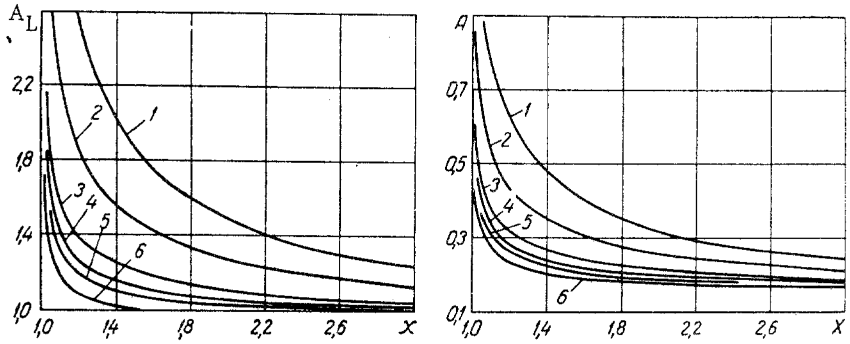

Fig. 1. Graphs of the functions $A_L(K_v, x)$, $A(K_v, X)$: $1 - K_v = 0.5$, $2 - 1$, $3 - 3$, $4 - 6$, $5 - 10$, $6 - \infty$

Results are given in [9] of measurements of the average heat transfer coefficient for a disc rotating in a thermally insulated housing with a radial flow. For a disc 45 mm thick, over which the temperature distribution was quadratic, the experimental data fitted the generalized relation ($Pr = 0.72$):

$$Nu = 0.0346\, Re^{0.8} K_v^{-0.1} X^{-0.3} (s/R)^{0.06}. \tag{13}$$

In the experiments the independent variables were varied within the limits $5 \cdot 10^5 \le Re \le 4 \cdot 10^6$; $0.6 \le K_v \le 7.0$; $2.15 \le X \le 2.7$; $0.016 \le s/R \le 0.065$. According to (5), the maximum boundary-layer thickness at the disc is then ~ 10 mm, while the boundary-layer thickness at the housing, computed from the momentum equation in the radial direction, exceeds 20 mm. Therefore, in making the comparison with the theoretical relation (11), in order to use the assumption of inviscid core flow, s in (13) is taken to be 30 mm. Then, for $R = 307.5$ mm, we have

$$Nu = 0.0303\, Re^{0.8} K_v^{-0.1} X^{-0.3}. \tag{14}$$

We note that the exponents of K_v and X in (13) were determined from the results of two series of tests, in which one of these parameters was kept fixed. Thus the exponent of K_v was determined from experiments, in which K_v was varied over the range 0.6-7.0 with $X = 2.7$ throughout, while the exponent of X was determined with $K_v = 2.0$ throughout, X being varied over the range 2.15-2.7 (two points). Approximating the graphs of Fig. 2 by means of a power law, we have

$$\varepsilon = 1.015 K_v^{0.1} X^{0.1}. \tag{15}$$

The error in the approximation for $X = 2.7$; $0.9 \le K_v \le 7$ is 2%, and for $K_v = 2.0$, $2 \le X \le 3$ it is 0.6%. In the quadrilateral $2 \le X \le 3$, $0.9 \le K_v \le 7.0$, the maximum error reaches 5% (in the region $x = 2$, $K_v = 0.9$).

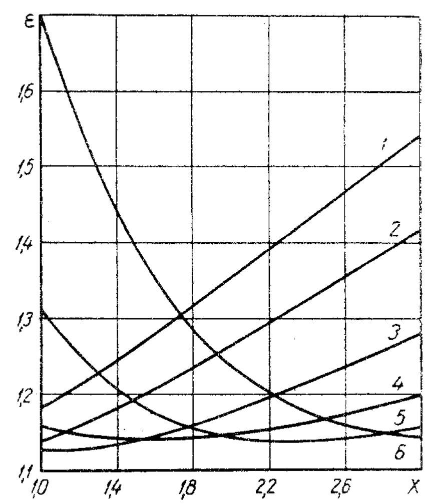

Fig. 2. Function ε (K_v, x): $1 - K_v = 10$, $2 - 6$, $3 - 3$, $4 - 2$, $5 - 1$, $6 - 0.5$

Combining (12) and (15), and taking into account the influence of Pr number by introducing the factor $Pr^{0.6}$ as for a freely rotating disc [8], we have

$$Nu = 0.0318\, Re^{0.8} K_v^{-0.1} X^{-0.3}. \tag{16}$$

In this case in (12) we put $(X^{5.25} - 1)^{0.8} X^{-0.6} (X^4 - 1)^{-1} \approx X^{-0.4}$. The discrepancy between the calculated and experimental relations (14) and (16), taking into account the error in approximating to ε, does not exceed 10%.

Analysis of the graphs in Figs. 1 and 2 shows that, in general, the exponents of K_v and X will vary. In the limiting cases of $K_v \to 0$, $K_v \to \infty$ when $x^4 \gg 1$, we have, from (11)

$$Nu = 0.050\, Re^{0.8} K_v^{-0.8} X^{1.6} = 0.050 \left(\frac{Q}{2\pi s \nu}\right)^{0.8}. \quad Nu = 0.023\, Re^{0.8}$$

(the last formula is the same as the solution obtained in [7] for a freely rotating disc.) Thus the exponent of K_v will vary from -0.8 to 0 over a wide range of variation in flow. This has been confirmed by mass transfer tests on a disc with radial flow [10].

For a small flow of coolant (large K_v), there is a rapid growth in the boundary-layer thickness at disc and housing. Therefore, beginning at the section where the region of viscous disturbance becomes equal to the gap width, the heat transfer coefficient should be calculated by the method of [11], which assumes that the boundary layers at disc and housing meet.

In conclusion, it should be noted that, in the case of arbitrary temperature distribution over the radius of the disc, the formulas for the local and average heat transfer coefficients may be obtained on the basis of the particular solution examined by the method discussed in [12]. The corresponding computations as applied to a disc are given in [1].

NOTATION

V_φ, V_r – circumferential and radial velocity components; r, R – variable and maximum radius of disc; z – axial coordinate, normal to surface of disc; r_0 – radius of coolant inlet; Q – volume flow of coolant; ω – angular velocity of disc; s – gap width between disc and housing; δ – thickness of hydrodynamic boundary layer; τ_φ – circumferential friction stress; V_0 – resultant relative velocity of flow around disc; β – angle between V_0 and circumferential direction; t_0 – initial temperature of coolant; t_d – disc temperature; Nu_L, Nu – local and average Nusselt number; Re_L, Re – local and average Reynolds number; $x = r/r_0$, $X = R/r_0$.

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Lenin Polytechnic Institute, Kharkov.